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A. H. Bell, Hillsboro, Ill., finds the 13 sets of values, and deduces the general rule for finding the same from the principle shown in his solution of Problem 31, page 363, Vol. II, of Monthly. P. S. Berg, A. M., Principal of Schools, Larimore, N. D., sends a good solution finding the 13 sets of values by the method of solution No. I, of the problem last referred to.

William Hoover, A. M., Ph. D., Ohio University, Athens, Ohio, solves substantially as in Solution I, above, but as he did not carry out the work far enough to give all the results, we adopted the phraseology of the Proposer.

Also excellently solved, but without determining the full sets of results, by Otto Clayton, A. B., Remington, Ind.; J. O. Mahoney, B. E., M. Sc,. Lynnville, Tenn.; and O. W. Anthony, M. Sc., Instructor in Mathematics in Boys' High School, New York City.

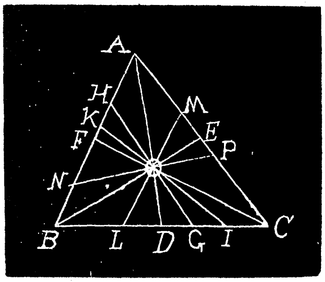
AVERAGE AND PROBABILITY.

56. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Find the chance that the center of gravity of a triangle lies inside the triangle formed by three points taken at random within the triangle. [From *Williamson's Integral Calculus.*]

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.

Let D, E, and F be the middle points of the respective sides of the triangle. Then O, the common intersection of HD, BE, and CF, is the center of gravity of the triangle. Suppose that the first point is somewhere upon the line OI, and the second somewhere upon the line OG. Put $DI=x$ and $DG=y$. Then the favorable positions for the third point are upon the surface OKH, whose area is



$$\frac{\triangle}{3}\left(\frac{4x}{a+6x}-\frac{4y}{a+9y}\right)$$

y being less than x, and x less than $\frac{1}{2}a$.

If $LD=y$, and the second point be upon OL, the favorable positions for the third point are upon the surface $OKAMO$, whose area is

$$\frac{\triangle}{3}\left(\frac{4x}{a+6x}+\frac{4y}{a+6y}\right).$$

If $FN=y$ and the second point be upon the line ON, the favorable positions for the third point are upon the surface $OKAPO$, whose area is

$$\frac{\triangle}{3}\left(\frac{4x}{a+6x}+1-\frac{4y}{a+6y}\right).$$

If G be taken between I and C, H will fall between F and K, and the favorable positions for the third point will be upon OHK, whose area is

$$\frac{\triangle}{3}\left(\frac{4y}{a+6y}-\frac{4x}{a+6x}\right).$$

The chance that the first point falls upon the element of surface at OI is $(\triangle/3)(dx/a)/\triangle=dx/3a$.

The chance that the second falls upon the element of surface at OG or OL is $dy/3a$. The chance that it falls upon the element at ON is $dy/3c$.

The chance that the third point falls upon any surface S is $S/\triangle$.

Hence if the first point be confined to the surface DOC and the second to surface CFB, the required chance is

$$P_1=\frac{1}{27a^2}\left[\int_0^{\frac{1}{2}a}\int_0^{x}\left(\frac{4x}{a+6x}-\frac{4y}{a+6y}\right)dxdy+\int_0^{\frac{1}{2}a}\int_0^{y}\left(\frac{4y}{a+6y}-\frac{4x}{a+6x}\right)dydx\right.$$
$$\left.+\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}a}\left(\frac{4x}{a+6x}+\frac{4y}{a+6y}\right)dxdy\right]+\frac{1}{27ac}\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}c}\left(\frac{4x}{a+6x}+1-\frac{4y}{c+6y}\right)dxdy.$$

By combining symmetrical expressions this reduces to

$$P_1=\frac{8}{27a^2}\left[\int_0^{\frac{1}{2}a}\int_0^{x}\left(\frac{x}{a+6x}\right)dxdy-{}^*\int_0^{\frac{1}{2}a}\int_0^{y}\left(\frac{x}{a+6x}\right)dydx\right.$$
$$\left.+\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}a}\left(\frac{x}{a+6x}\right)dxdy\right]+\frac{4}{27ac}\left[\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}c}\left(\frac{x}{a+6x}\right)dxdy\right.$$
$$\left.-\int_0^{\frac{1}{2}c}\int_0^{\frac{1}{2}a}\left(\frac{y}{c+6y}\right)dydx+\tfrac{1}{4}\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}c}dxdy\right]$$

$$=\frac{8}{27a^2}\left[\int_0^{\frac{1}{2}a}\left(\frac{x^2}{a+6x}\right)-\frac{x(\frac{1}{2}a-x)}{a+6x}+\tfrac{1}{2}a\left(\frac{x}{a+6x}\right)\right]$$
$$+\frac{4}{27ac}\left[\int_0^{\frac{1}{2}a}\tfrac{1}{2}c\left(\frac{x}{a+6x}\right)dx-\int_0^{\frac{1}{2}c}\tfrac{1}{2}a\left(\frac{y}{c+6y}\right)dy+\tfrac{1}{2}c\int_0^{\frac{1}{2}a}dx\right]$$

$=\frac{13}{972}+\frac{4}{729}\log2$. Since this is independent of a, b, and c, it is evident that if the second point had been allowed to occupy all positions on both sides of CF, the result would have been twice as great, and if the first point had been allowed to take all positions instead of being confined to DOC the result would have been six times as great.

Hence the required probability is $P=12P_1=\frac{13}{81}+\frac{16}{243}\log2=\frac{1}{81}[13+\frac{16}{3}\log2]$ $=.20613$.

Remark. In integrating the expression marked *, I reversed the order of integration. Thus,

$$\int_0^{\frac{1}{2}a}\int_0^{y}\left(\frac{x}{a+6x}\right)dydx=\int_0^{\frac{1}{2}a}\int_x^{\frac{1}{2}a}\left(\frac{x}{a+6x}\right)dxdy=\int_0^{\frac{1}{2}a}\left(\frac{x(\frac{1}{2}a-x)}{a+6x}\right)da.$$

[The result given in *Williamson's Calculus* is $\frac{1}{27}(2+\frac{10}{3}\log4$. Editor.]